

Notes on covariant quantities in noninertial frames and invariance of radiation in classical and quantum field theory

Hrvoje Nikolić

Theoretical Physics Division, Rudjer Bošković Institute,

P.O.B. 1016, HR-10001 Zagreb, Croatia

hrvoje@faust.irb.hr

February 7, 2008

Abstract

A local observer can measure only the values of fields at the point of his own position. By exploring the coordinate transformation between two Fermi frames, it is shown that two observers, having the same instantaneous position and velocity, will observe the same values of covariant fields at their common instantaneous position, even if they have different instantaneous accelerations. In particular, this implies that in classical physics the notion of radiation is observer independent, contrary to the conclusion of some existing papers. A “freely” falling charge in curved spacetime does not move along a geodesic and therefore radiates. The essential feature of the Unruh effect is the fact that it is based on a noninstantaneous measurement, which may also be viewed as a source of effective noncovariance of measured quantities. The particle concept in Minkowski spacetime is clarified. It is argued that the particle concept in general spacetime does not depend on the observer and that there exists a preferred coordinate frame with respect to which the particle number should be defined.

PACS number(s): 04.20.Cv, 03.50.-z, 41.60.-m, 04.62.+v

1 Introduction

As is well known, an accelerated charge in flat Minkowski spacetime radiates, at least as seen by an inertial observer. This naturally raises the following questions: Does an inertially moving charge radiate from the point of view of an accelerated observer? Does an accelerated charge radiate from the point of view of an observer who also accelerates with the same acceleration? Can one generalize the answers to these questions to curved spacetime and

gravitational acceleration, only using the equivalence principle? The first aim of this paper is to give the answers to these and some related questions in the framework of classical physics.

These questions have already been discussed in several papers [1-9]. In [5, 9] the fields of an inertial charge in flat spacetime are transformed to Rindler coordinates, in which the Poynting vector $\mathbf{S} = \mathbf{E} \times \mathbf{B}$ does not vanish everywhere. It is therefore concluded that an inertially moving charge radiates from the point of view of an accelerated observer. Similarly, by transforming the fields of a uniformly accelerated charge expressed in Minkowski coordinates to Rindler coaccelerating coordinates, it is found that the Poynting vector vanishes at all points to which Rindler coordinates can be applied. It is therefore concluded in [1, 3, 5, 6, 9] that the accelerated charge does not radiate from the point of view of a coaccelerating observer, although it does radiate from the point of view of an inertial observer. In [1, 6, 8] it is also argued that a noninertial static charge in a static gravitational field does not radiate from the point of view of a static observer, because (since everything is static in the corresponding coordinates) the fields must be time independent.

We criticize such a reasoning. For a given coordinate frame, the values of covariant fields are formally determined at all spacetime points covered by these coordinates. However, a local observer can measure only the values of fields at the point of his own position. It is completely unphysical to talk about the value of a field at some point from the point of view of the observer sitting at some other point. Rindler coordinates are a special case of Fermi coordinates. By exploring the coordinate transformation between two Fermi frames, we show that two observers, having the same instantaneous position and velocity, will observe the same values of covariant fields at their common instantaneous position, even if they have different instantaneous accelerations. When radiation is defined in an operationally meaningful way, this fact leads to the conclusion that radiation does not depend on the observer, but only on the motion of the radiating charge.

We also study the concept of particle in quantum field theory in flat and curved spacetime. It is often argued (originally in [10]) that a response of an accelerated “particle detector” can be interpreted as a dependence of the particle number on coordinates. Although one could even accept that Minkowski coordinates have a privileged role in flat spacetime, it is usually argued that there is no such privileged coordinate frame in general curved spacetime. In this way the concept of particle becomes very fuzzy and, in particular, observer dependent.

We argue, both formally and operationally, that the number of particles in a given quantum state does *not* depend on the observer. We give an invariant operational meaning to the particle number by discussing the response of classical detectors, such as a Wilson chamber or a Geiger-Müller counter. We also argue that quantum physics and cosmological observations imply that a preferred coordinate frame *does* exist, which leads to a preferred representation of the algebra of quantum fields, but does not violate the covariance. This preferred representation leads to an unambiguous formal definition of the particle number.

The paper is organized as follows: In Sec. 2 we explain the physical meaning of Fermi coordinates and study the coordinate transformation between two Fermi frames. In Sec. 3 we give a general discussion on measurable quantities in classical field theory. The concept of classical radiation is discussed in Sec. 4, where it is also shown that a “freely” falling charge in curved spacetime does not move along a geodesic and consequently radiates. In Sec. 5 we argue that in quantum field theory the measurable quantities do not need to transform covariantly, which illuminates the origin of the Unruh effect. We also argue that

the essential property of the measurements that can lead to the Unruh and similar effects is a large duration of the measuring procedure. The concept of particle in Minkowski spacetime is clarified in Sec. 6. Section 7 is devoted to the concept of particle in general spacetime, while Sec. 8 is devoted to the concluding remarks.

We use the signature $(+---)$ and the units $\hbar = c = k_B = 1$, where k_B is the Boltzmann constant.

2 Coordinate transformation between two Fermi frames

Before starting with calculations, we clarify the physical meaning of Fermi coordinates, reviewing some results already presented in [11]. Fermi coordinates are proper coordinates of a local observer determined by its trajectory in spacetime. It is convenient to define them such that the observer is positioned at the space origin. Therefore, the metric expressed in Fermi coordinates has the property $g_{\mu\nu}(t, 0, 0, 0) = \eta_{\mu\nu}$ [12]. Even if there is no relative motion between two observers, they belong to different Fermi frames if their trajectories do not coincide. However, for practical purposes, they can be considered as belonging to the same frame if there is no relative motion between them and the other observer is close enough to the first one, in the sense that the metric expressed in Fermi coordinates of the first observer does not depart significantly from $\eta_{\mu\nu}$ at the position of the second one. In particular, this implies that two inertial observers in flat spacetime who have the same velocity can always be considered to be in the same frame, no matter how distant they are from each other. In other words, Fermi coordinates have a clear physical interpretation only in a small neighborhood of the physical observer to whom the Fermi coordinates refer. It is an exclusive property of Minkowski coordinates, among other inertial and noninertial Fermi coordinates, to have a clear global physical interpretation. Since the intuition of physicists is often based on the familiarity with Minkowski coordinates, this is the source of many misinterpretations. Some of them have been discussed in [11], while some others are discussed in this paper.

We first consider the coordinate transformation between different Fermi frames in flat spacetime. Let S be an inertial frame and let S' be the frame of the observer whose 3-velocity is $u^i(t') \equiv \mathbf{u}(t')$, as seen by an observer in S . In general, S' can also rotate, which can be described by the rotation matrix $A_{ji}(t') = -A_j^i(t')$. The coordinate transformation between these two frames can be written as [11]

$$x^\mu = \int_0^{t'} f_0^\mu(t', 0; \mathbf{u}(t')) dt' + \int_C f_i^\mu(t', \mathbf{x}'; \mathbf{u}(t')) dx'^i, \quad (1)$$

where

$$f_\nu^\mu = \left(\frac{\partial f^\mu}{\partial x'^\nu} \right)_{\mathbf{u}=\text{const}}, \quad (2)$$

and

$$x^\mu = f^\mu(t', \mathbf{x}'; \mathbf{u}) \quad (3)$$

denotes the ordinary Lorentz transformation, i.e., the transformation between two inertial frames specified by the constant relative velocity \mathbf{u} . In (1), C is an arbitrary curve with

constant t' , starting from 0 and ending at $-A_j^i(t')x'^j$. It is assumed that the space origins of S and S' coincide at $t = t' = 0$. The coordinates x^μ are the Fermi coordinates of the observer positioned at $x'^i = 0$. By combining two transformations of the form (1) one can find the transformation between two noninertial Fermi frames. The quantity $f_\nu^\mu = \partial x^\mu / \partial x'^\nu$, which is relevant to the transformation of tensors, is, in general, a complicated function of x' . However, if one restricts the analysis to the point $x'^i = 0$, one obtains much simpler relations.

Let us calculate f_ν^μ at $x'^i = 0$ for a *fixed* instant t' . Without losing on generality, we choose the rotation matrix such that $A_{ij}(t') = \delta_{ij}$, which means that the corresponding space axes of S and S' are parallel at this particular instant. From (1) and the well-known Lorentz transformations in (3) one easily finds

$$\begin{aligned} f_0^0 &= \gamma, & f_j^0 &= -\gamma u_j, & f_0^i &= \gamma u^i, \\ f_j^i &= \delta_j^i + \frac{1 - \gamma}{\mathbf{u}^2} u^i u_j, \end{aligned} \quad (4)$$

where $u^j = -u_j$, $\mathbf{u}^2 = u^i u^i$, $\gamma = 1/\sqrt{1 - \mathbf{u}^2}$ are evaluated at t' . We see that f_ν^μ at the position of the observer in S' depends only on the instantaneous velocity and that this dependence is the same as for two inertially moving observers. Actually, in a more general case there is also a dependence on the instantaneous orientation of the space axes, but this is physically irrelevant.

Let us now generalize these results to curved spacetime. Let S and S' be the Fermi frames of two arbitrarily moving observers. Assume that they have the same position at $t = t' = 0$. This implies that the coordinate transformation between the two coordinate frames takes the form

$$x^\mu = f_\nu^\mu x'^\nu + f_{\nu\alpha}^\mu x'^\nu x'^\alpha + \dots \quad (5)$$

We are interested only in the quantity

$$f_\nu^\mu = \left(\frac{\partial x^\mu}{\partial x'^\nu} \right)_{x=x'=0}. \quad (6)$$

The metric tensor at $x = x' = 0$ transforms as

$$g'_{\mu\nu} = f_\mu^\alpha f_\nu^\beta g_{\alpha\beta}. \quad (7)$$

Since we study the Fermi coordinates, the metric has a form $g_{\alpha\beta}(x) = \eta_{\alpha\beta} + \mathcal{O}(x^i)$, $g'_{\mu\nu}(x') = \eta_{\mu\nu} + \mathcal{O}(x'^i)$. Putting this in (7), we obtain the equation

$$\eta_{\mu\nu} = f_\mu^\alpha f_\nu^\beta \eta_{\alpha\beta}. \quad (8)$$

Therefore, the problem of finding the coefficients f_ν^μ reduces to the problem of finding the coefficients of a linear transformation $x^\mu = f_\nu^\mu x'^\nu$ that preserves the Minkowski metric $\eta_{\mu\nu}$. This is nothing else but how the ordinary Lorentz transformations are usually found. The solution of this problem is well known and is given by (4), where $u^i = dx^i/dt$ is the velocity of the observer in S' as seen by the observer in S , at the instant when the two observers have the same position. Of course, one can generalize (4) for the case when one of the observers

rotates relative to the other, but this is again physically irrelevant, because one can always choose the space axes of the two frames such that the corresponding axes are parallel at the instant of interest.

The physical consequence of the result obtained in this section is the following: Let $\Phi_{\alpha_1 \dots \alpha_n}(x)$ be an arbitrary local tensor quantity. Let the two observers measure this quantity at their common instantaneous position. The results of measurements will be related as

$$\Phi'_{\mu_1 \dots \mu_n} = f_{\mu_1}^{\alpha_1} \dots f_{\mu_n}^{\alpha_n} \Phi_{\alpha_1 \dots \alpha_n} . \quad (9)$$

The two measurements will be different if there is an instantaneous relative velocity between the two observers. However, the instantaneous relative acceleration, as well as higher-order derivatives, are irrelevant to this transformation law.

Equation (9) is the local transformation law of covariant tensors. One can easily find the generalization for contravariant or mixed tensors, by performing contractions with $\eta^{\mu\nu}$.

3 What is measurable in classical field theory?

In this section we explain in more detail the local nature of classical measurements.

A local observer can measure only the values of fields at the point of his own position. It is completely unphysical to talk about the value of a field at some point from the point of view of the observer sitting at some other point. For example, when somebody “sees” a distant object, he actually measures the properties of reflected light at his own position. Since all the interactions are local, a measuring apparatus can only response to the values of fields at the position of the apparatus.

In practice, a measuring apparatus is usually a large object. In this case, each part of the apparatus should be regarded as a separate local measuring device, each having its own Fermi coordinates. The response of the apparatus as a whole is some kind of sum or average of the responses of all its parts, where the interaction between various parts of the apparatus may also play a significant role. In this sense, a measurement made using a large apparatus can be regarded as an unideal measurement. Some similar remarks have already been given in [7], but the consequences of such a reasoning have not been explored completely.

To calculate how a large measuring apparatus will respond to a given field at a certain instant (or, better to say, at a certain spacelike hypersurface), one needs to specify the velocity of each part of the apparatus at this instant. If the measured field transforms as a tensor, the instantaneous accelerations are irrelevant.

A measurement can also be unideal if it lasts a finite time. Obviously, if two observers have equal velocities initially, but different velocities later, a cumulative effect, which depends on all instantaneous velocities, will be different for the two observers. However, it is important to realize that the acceleration itself is not essential for understanding such effects. (An example of such an effect is the twin paradox, where, contrary to the common belief, the acceleration is not essential. For example, in curved spacetime it may be possible to connect two points by two different timelike geodesics that have different proper lengths. If these two geodesics correspond to the trajectories of two observers, one will obtain the twin “paradox” without acceleration.)

In practice, no measurement is ideal. However, we can introduce the concept of almost ideal measurements, i.e., the measurements that last short enough such that the velocity and the measured field do not change significantly and are performed using a small apparatus such that the metric of the corresponding Fermi coordinates and the measured field do not vary significantly through the apparatus.

4 What is radiation in classical electrodynamics?

In this section we attempt to give an operational and therefore physically meaningful definition of the concept of radiation in classical electrodynamics, based on ideal or almost ideal measurements.

In [2] the measure of radiation is defined in a Lorentz invariant manner, as the flux of the Poynting vector through a closed two-dimensional surface that looks as a sphere for an inertial observer instantaneously comoving with the charge at the retarded time. It is also shown that the radius of this sphere does not need to be large. We call this the standard definition of radiation.

However, such a definition of radiation is global. We need criteria that can answer the question whether there is a radiation at a certain spacetime point. We also require that it should be possible to apply the criteria to inertial as well as to accelerated observers, in flat as well as in curved spacetime.

There is an attempt [4] to show that an inertial charge in flat spacetime radiates from the point of view of an accelerated observer, by studying the response of a specific model of a classical detector. It is shown that this detector absorbs energy when accelerates uniformly, but does not absorb energy when moves inertially. However, the behavior of this specific unideal “detector” proves nothing, because one can easily model a detector that will absorb energy even when moves inertially. If one wants to see whether the concept of radiation depends on the motion of the observer, the answer should not depend on details of the measuring apparatus.

We note that radiation is not a kinematical effect resulting from the coordinate transformation between the frames of the radiating charge and the observer, but a dynamical effect, in the sense that even for the observer comoving with the charge, the fields depend on acceleration. This is not explicitly seen in the conventional approach in which the Maxwell equations are solved in Minkowski coordinates. To see this explicitly, we write the covariant Maxwell equation

$$D_\mu F^{\mu\nu} = j^\nu \quad (10)$$

in a more explicit form

$$\partial_\mu F^{\mu\nu} + \Gamma_{\mu\lambda}^\mu F^{\lambda\nu} + \Gamma_{\mu\lambda}^\nu F^{\mu\lambda} = j^\nu. \quad (11)$$

We assume that the current j^ν corresponds to a pointlike charge. Let us study how the field $F^{\mu\nu}$ looks like to an observer comoving with the charge in his small neighborhood. Since he uses the corresponding Fermi coordinates, the connections $\Gamma_{\beta\gamma}^\alpha$ vanish in his small neighborhood if and only if his trajectory is a geodesic [12]. Therefore, if the charge does not accelerate, in the small neighborhood of the charge the solution of (11) looks just like the well-known Coulomb solution $\mathbf{E} \propto r^{-2}$, $\mathbf{B} = 0$. On the other hand, if the charge accelerates,

then, even in the small neighborhood, Eqs. (11) no longer look like the Maxwell equations in Minkowski spacetime. This gives rise to a more complicated solution, which includes the terms proportional to r^{-1} . A similar approach to radiation has been studied in [13]. We can apply the standard definition of radiation to the small neighborhood of the charge and conclude in this way that the accelerated charge always radiates and the inertial charge never does, even in curved spacetime. In this way, the radiation is an intrinsic property of the fields in the small neighborhood of the radiating charge, seen by a comoving observer. If this intrinsic definition of radiation is accepted, then, obviously, radiation does not depend on the observer.

Once the fields leave its source, their propagation is determined only by the spacetime geometry. Therefore, one should be able to answer the question whether the charge radiates by measuring the fields at arbitrary distances from the charge. However, the conclusion should not depend on the observer.

One could try to propose that at a certain spacetime point there is radiation if the Poynting vector does not vanish at this point. However, even two inertially moving observers in flat spacetime may not agree on whether the Poynting vector vanishes at a certain point.

One could propose that “to radiate” means “to emit energy”. However, in non-Minkowski spacetime, the global concept of energy is not well defined. One could use our local philosophy to note that in the vicinity of any observer the metric is Minkowskian, so the energy that he can measure is well defined. However, there is a trouble again, because, even in globally Minkowskian spacetime, the energy-momentum tensor is defined up to a total derivative, which does not affect the total energy, but does affect the energy contained in a small volume. Nevertheless, one can choose some specific definition of the energy-momentum tensor, which, in our opinion, leads to the best possible definition of energy, namely, the energy contained in a small volume measured by a local observer. All other definitions of energy, such as the global energy based on a timelike Killing vector (if it exists), are unoperational and therefore unphysical.

The only measurable entity related to the electromagnetic field is its effect on charges, described by the covariant equation

$$m\ddot{x}^\mu = q F_{\text{ext}}^{\mu\nu} \dot{x}_\nu + \Gamma^\mu, \quad (12)$$

where

$$\Gamma^\mu = \frac{2}{3} q^2 [\ddot{v}^\mu - v^\mu \dot{v}_\alpha \dot{v}^\alpha] + \Gamma_{\text{curv}}^\mu \quad (13)$$

and the quantities

$$\begin{aligned} v^\mu &= \dot{x}^\mu = dx^\mu/d\tau, \\ \dot{v}^\mu &= \ddot{x}^\mu = d\dot{x}^\mu/d\tau + \Gamma_{\beta\gamma}^\mu \dot{x}^\beta \dot{x}^\gamma, \\ \ddot{v}^\mu &= d\ddot{x}^\mu/d\tau + \Gamma_{\beta\gamma}^\mu \ddot{x}^\beta \dot{x}^\gamma \end{aligned} \quad (14)$$

transform as vectors [14]. In (12) the first term represents the familiar Lorentz force caused by an external field $F_{\text{ext}}^{\mu\nu}$. The Γ^μ term represents the self-force, where the first term in (13) is the well-known Abraham-Lorentz force. The Γ_{curv}^μ term depends on the curvature and vanishes when the curvature is zero, but for the nonvanishing curvature it is nonvanishing

even when the acceleration \dot{v}^μ and the higher derivatives are zero. The explicit form of Γ_{curv}^μ is given in [15, 16].

There is a simple intuitive picture explaining why a self-force appears when the charge accelerates or when spacetime around the charge is curved. The fields produced by the charge always act on it. However, when the charge moves inertially through flat spacetime, then the metric related to the corresponding Fermi coordinates is isotropic, and so are the fields. This implies that the self-forces in different directions cancel exactly, so the resultant force is zero. When spacetime is curved (such that it is not isotropic) or the charge accelerates, then the metric related to the Fermi coordinates is no longer isotropic. Consequently, the fields are also not isotropic, which implies that the resultant force need not to be zero.

The presence of the Γ_{curv}^μ term in (13) implies that even in the absence of external electromagnetic fields, a charge in curved spacetime does not move geodesically. This is not inconsistent with the equivalence principle, because it states that when *only* gravitational fields act on a pointlike particle, then the motion of the particle does not depend on internal properties of the particle (such as its charge). In our case, there are also electromagnetic forces produced by the charge. Alternatively, one can regard the charge and its fields as one object, which is no longer pointlike, so the assumptions of the equivalence principle are violated again.

Since a “freely” falling charge in curved spacetime does not move geodesically, it follows, according to our intrinsic definition of radiation and Eqs. (11), that it radiates. Of course, in curved spacetime one does not expect, in general, that radiating fields will fall off as r^{-1} at large distances (it is also not clear what r is in curved spacetime). However, it is reasonable to expect that at large distances the fields produced by accelerated (i.e., nongeodesically moving) charges will be much stronger than those of geodesically moving charges.

Now we turn back to the attempt to give an operational definition of radiation at large distances. In our opinion, the only reason why radiating fields deserve special attention in physics, is the fact that they fall off much slower than other fields, so their effect is much stronger at large distances. Actually, the distinction between “radiating” and “nonradiating” fields is quite artificial; there is only one field, which can be written as a sum of components that fall off differently at large distances. If one knows the distance of the charge that produced the electromagnetic field $F_{\text{ext}}^{\mu\nu}$ and measures the intensity of its effects described by (12), then one can determine whether this effect is “large” or “small”, i.e., whether the charge radiates or not. If the field is proportional to r^{-1} as seen by one observer, it is also so as seen by any other observer at the same position. Since $F_{\text{ext}}^{\mu\nu}$ in (12) transforms as a tensor, the effect may depend on the velocity of the observer, but cannot depend on its acceleration. In this sense, we can say that radiation does not depend on the observer.

To conclude this section, we note that the concepts of energy and radiation in classical field theory are only auxiliary concepts. These concepts may be useful and well defined in Minkowski spacetime, but not in general. All that one really needs is contained in unambiguous equations of motions, such as (10) and (12). However, for those who insist on giving a definition of these quantities in general spacetime, in this section we have attempted to give the best possible operational definitions. One may feel that these definitions are far from being precise, but this is because we have attempted to define concepts which are not really meaningful at the fundamental level. Nevertheless, it is our conclusion that the best possible definition of radiation leads to the conclusion that the charge “radiates” if and only

if it does not move geodesically. It does not depend on the observer.

5 Noncovariant quantities, time nonlocal measurements, and quantum field theory

If a measurable quantity does not transform covariantly, i.e., as a tensor, then the transformation law (9) cannot be applied. This fact can be particularly illuminating for understanding some effects of quantum field theory. For example, if one does not use the normal ordering of operators, then the expectation value of the energy-momentum tensor can always be written as

$$\langle \psi | T^{\mu\nu} | \psi \rangle = t_{\text{inf}}^{\mu\nu} - t_{\text{meas}}^{\mu\nu} , \quad (15)$$

where $t_{\text{inf}}^{\mu\nu}$ is an infinite unmeasurable part, whereas $t_{\text{meas}}^{\mu\nu}$ is the measurable part. (The minus sign in (15) is a matter of convenience; one can also obtain a positive sign by redefining the infinite part.) The left-hand side formally transforms covariantly, but it is infinite and therefore cannot be measurable. The measurable part depends on details of the measuring procedure. Accordingly, there is no *a priori* reason why the two terms on the right-hand side should transform separately as tensors.

The Unruh effect can be viewed as an example of such an effect. If $|\psi\rangle = |0\rangle$, then $t_{\text{meas}}^{\mu\nu} = 0$ for an inertial observer. On the other hand, an observer accelerated uniformly with the acceleration a measures a thermal energy-momentum tensor corresponding to the temperature $T = a/2\pi$, which does not vanish. Obviously, the measurable part of $\langle 0 | T^{\mu\nu} | 0 \rangle$ does not transform covariantly. To see more closely how this happens, consider a real massless scalar field in $1 + 1$ dimensions quantized using the standard Minkowski quantization. We study the correlation function

$$\Gamma(x_i, x_f) = \frac{1}{4} \langle 0 | \partial_0 \phi(x_i) \partial_0 \phi(x_f) + \partial_1 \phi(x_i) \partial_1 \phi(x_f) + (x_i \leftrightarrow x_f) | 0 \rangle , \quad (16)$$

where $x_i = x(\tau_i)$, $x_f = x(\tau_f)$, and $x(\tau)$ is the trajectory of the observer. The partial derivatives are calculated with respect to the Fermi coordinates of the observer. This correlation function has the property

$$\langle 0 | S T^{00}(x) | 0 \rangle = \Gamma(x, x) , \quad (17)$$

where S denotes the symmetric ordering. Using the results of [17], we can immediately write (16) for a uniformly accelerated observer. It appears that Γ depends only on $\Delta\tau = \tau_f - \tau_i$ and can be written as

$$\Gamma(\Delta\tau) = \frac{1}{(\Delta\tau)^2} - \left[\frac{1}{(\Delta\tau)^2} - \frac{a^2}{4 \text{sh}^2(a \Delta\tau/2)} \right] . \quad (18)$$

The first term, infinite in the limit $\Delta\tau \rightarrow 0$, corresponds to that which would be obtained for an inertial observer. The term in the square brackets vanishes for $\Delta\tau \rightarrow 0$ and has a thermal Fourier transform [17]. This suggests that the first term should be interpreted as the infinite unmeasurable term, while the term in the square brackets is the finite measurable one. This also demonstrates that this noncovariant effect disappears when the measurement is instantaneous, i.e., $\Delta\tau \rightarrow 0$.

Note that the breaking of scale invariance through the renormalization in quantum field theory, well known by particle physicists, can also be viewed in a similar way. An infinite, but scale-invariant correlation function obtained before the renormalization can be written as a sum of two scale-noninvariant terms, one of them being the finite measurable term.

To understand what is measurable and how the result depends on the duration of the measurement, one needs to study a model of the detector. In a simple pointlike “particle detector” moving arbitrarily in flat spacetime and initially being in its ground state E_0 , the amplitude for the transition to the excited state E is [18] (see also [19] for a justification of such a simple model)

$$A(|0\rangle \rightarrow |k\rangle, E_0 \rightarrow E) \propto \int_{\tau_i}^{\tau_f} d\tau e^{i(E-E_0)\tau} \langle k | \phi(x(\tau)) | 0 \rangle, \quad (19)$$

where it is assumed that the field $\phi(x)$ is initially in its vacuum state $|0\rangle$ and $\tau_f - \tau_i$ is the duration of the measuring procedure. The final state of the field is a one-particle state $|k\rangle$. In this model, the detector can spontaneously jump into its excited state, owing to vacuum fluctuations. Note that this amplitude is nonvanishing even for an inertially moving detector. Only if one takes $\tau_f - \tau_i = \infty$ (as is usual in quantum mechanics), one obtains a vanishing amplitude for an inertial detector and a thermal amplitude for a uniformly accelerating one. The ultimate reason why the lowest energy state $|0\rangle$ appears as “nothing” to an inertial observer, although the energy of this state is actually infinite, is the fact that typical measurements in quantum mechanics usually last very long compared with the inverse energy differences measured. This is why one can say that the energy of vacuum is zero. One need not use the normal ordering in order to calculate the physical effects of various states. For example, when one uses the symmetric ordering of the Hamiltonian in perturbative calculations with infinite-time intervals and assumes that the field is in its vacuum state, one obtains two infinite terms that contribute to a spontaneous excitation of an atom in its ground state. However, these two terms cancel exactly for an inertial atom and lead to finite thermal excitations for a uniformly accelerating one [20].

The Unruh effect, viewed as an effect of vacuum fluctuations on a noninertially moving atom, is a quantum effect. However, it is important to note that the thermal nature of these excitations for a uniform acceleration can be understood even with classical physics [21]. Moreover, even the effect of stochastic quantum fluctuations on correlation functions can be simulated by viewing vacuum as a sea of classical random fluctuations with a Lorentz invariant spectrum [22].

It is also important to note that the actual response of a uniformly accelerating detector depends on details of interactions in the detector. In some cases, there is no response at all [23, 24].

6 The concept of particle in Minkowski spacetime

The Unruh effect is often interpreted as absorption of a “particle” which does not exist from the point of view of an inertial observer. Such an interpretation makes the concept of particle rather fuzzy. Before discussing the concept of particle in general spacetime in the next section, we find necessary first to clarify the concept of particle in an inertial, Minkowski frame.

An n -particle state is defined as a normalized state of the form

$$|n\rangle = \int d^3k_1 \cdots d^3k_n f(k_1, \dots, k_n) a^\dagger(k_1) \cdots a^\dagger(k_n) |0\rangle. \quad (20)$$

The crucial question we attempt to answer in this section is why such formally defined states correspond to that which is observed in typical experiments as n separated entities, i.e., “particles”.

We introduce the concept of a classical particle detector, such as a Wilson chamber or a Geiger-Müller counter. We call such detectors classical, because, in order to understand how and why such detectors respond, quantum field theory is not essential. Classical detectors respond to states that correspond to the classical concept of particle, i.e., to states which are well localized in space (small lumps of energy). For example, if two localized particles are very near each other, which can be achieved by a suitable choice of the function $f(k_1, k_2)$, a classical detector will see this state as one particle. However, it seems that such things do not occur in practice. Why?

Before answering this question, first consider how a one-particle state is detected. Assume that a particle detector is localized in space. The function $f(k_1)$ can correspond to a plane wave or to a state with two lumps, but such states cannot be observed by the localized detector. It will not be observed until the function $f(k_1)$ collapses to a one-lump state. (We do not attempt to answer the question whether the wave function collapses spontaneously or the detector causes it. In some interpretations of quantum mechanics there is a clear answer to this question, but we do not want to prefer any particular interpretation.)

Now consider a two-particle state. Assume that, *a priori*, all functions $f(k_1, k_2)$ are equally probable. However, the number of functions corresponding to a two-lump state is larger than that for a one-lump state, so it is very improbable that two particles will form one lump. (Of course, the numbers of two-lump states and one-lump states are both infinite, but their ratio is not equal to 1.) A four-lump state, for instance, is even more probable than a two-lump state, but if one particle is distributed in more than one lump, it will not be detected.

In this way we have explained why an n -particle state behaves as an n -lump state in experiments. Of course, if there are strong attractive forces among fields, there may exist a natural tendency that n -particle states form a one-lump state (hadrons, α -particles), in which case it is more convenient to treat such states as one-particle states. Actually, it is often completely incorrect to treat such states as n -particle states, because nonperturbative effects of interactions may completely change the spectrum of states of a free theory, such as is the case for QCD.

One may argue that classical detectors are not the best operational way to define a particle. One should rather study quantum detectors. For example, an atom absorbs precisely one photon, not a half of it, nor two of them. Even the response of a “classical” detector should be ultimately described in terms of virtual absorptions and emissions (this is the reason why a “classical” localized detector cannot respond to a two-lump one-particle state). However, it is important to emphasize that there is not any deep, fundamental principle which forbids absorption or emission of a half or two particles. It is merely a consequence of a particular form of dynamics. For example, an electron in the atom absorbs *one* photon because the interaction Lagrangian $\mathcal{L}_I = e\bar{\psi}\gamma_\mu\psi A^\mu$ is *linear* in A^μ . Of course, higher-order corrections

allow absorption of two photons (sum of their energies must be equal to the difference of energies of the atom levels), but such processes are suppressed dynamically (small coupling constant) and kinematically (small probability of a one-lump two-photon state). But is it, in principle, possible to absorb a half of a particle? One can exclude such a possibility if one proposes that the Lagrangian $\mathcal{L}(\phi, \partial_\mu \phi, \dots)$ of fundamental fields must be an analytic function around zero. For example, assuming that there are no derivative couplings, the analyticity implies that a local interaction Lagrangian has a form

$$\mathcal{L}_I(\phi(x), \dots) = \sum_{n \geq 0} C_n(x) (\phi(x))^n, \quad (21)$$

where n are integers and $C_n(x)$ depend on some other fields. The interaction (21) implies that the number of absorbed or emitted particles must be an integer. If $\phi(x)$ is an effective, composite field, like $\phi(x) = \chi(x)\chi(x)$, then one can have a term proportional to $\sqrt{\phi} = \chi$, in which case a “half” of the ϕ -particle, i.e., one χ -particle, can be absorbed or emitted. If one allows terms like $\sqrt{\phi}$ for fundamental fields, then even a “half” of a fundamental particle can be absorbed or emitted. (In this case, the concept of particle concept based on perturbative calculations is no longer a good concept. In particular, it is not clear, even algebraically, what $\sqrt{a^\dagger}|0\rangle$ is.)

7 The invariant concept of particle in general space-time

In the preceding section we have seen that even in Minkowski spacetime the concept of particle in quantum field theory is not completely clear. This is, of course, hardly surprising, because already in classical physics the concepts of a field and of a particle are quite different. It is usually argued that in non-Minkowski spacetime the concept of particle is even more fuzzy (see, for example, [18] and references therein) and, in particular, that the number of particles in a given state depends on the motion of the observer. In this section we argue that the concept of particle in general spacetime is *not* more problematic than in Minkowski spacetime.

One could be tempted to apply our local philosophy of Secs. 3 and 4, well suited for classical fields, to quantum fields as well. However, this would not be appropriate, because quantum physics possesses certain nonlocal properties, related to the uncertainty relations and EPR-like effects. Quantization requires a global approach, which is the reason that it is difficult to join quantum mechanics and general covariance.

It is often claimed (originally in [25]) that the Unruh effect, discussed in Sec. 5, can be alternatively described by the so-called Rindler quantization, which consists in decomposing free fields into modes proportional to $e^{\pm i\omega\tau}$, where τ is the time measured by a uniformly accelerated observer (this is the same time as t' in (1) for a uniform acceleration). The corresponding number of “Rindler particles” $N_R = \sum a_R^\dagger a_R$ in Minkowski vacuum $|0\rangle$ is not zero, but has a thermal distribution. Therefore, according to such an interpretation, the inertial and the accelerated observers do not agree on the number of particles contained in the state $|0\rangle$, and this is the reason that an accelerated atom can jump to an excited state.

However, contrary to the common belief, the two descriptions of the Unruh effect are *not* equivalent. In particular, the Rindler-quantization approach predicts that the absorption of a Rindler particle by the accelerated atom will be seen by an inertial observer as an emission of a Minkowski particle [25] only if the atom has actually jumped to the excited state [26]. On the other hand, by putting $E = E_0$ in (19), we see that the Minkowski-quantization approach predicts an emission of a Minkowski particle even if the transition to an excited state has not actually occurred. The fact that the two inequivalent approaches both lead to a thermal spectrum with the temperature $T = a/2\pi$ is hardly surprising, because, as we have already noted, classical physics also leads to a thermal spectrum with the same temperature [21, 22]. And even this partial agreement of the two approaches does not generalize when the uniform acceleration is replaced by a more complicated motion [27]. Since the two approaches are not equivalent, and since it is not reasonable to suspect that the predictions based on the Minkowski quantization are incorrect, one should reject the Rindler quantization as a correct description of nature.

Let us also discuss it from a more formal point of view. In particular, we are interested in the question whether the rejection of the Rindler and other “noninertial” quantizations is in contrast with the general covariance.

Note first that in classical physics the choice of modes for the field decomposition has nothing to do with the choice of coordinates. In order to describe classical effects as seen by a noninertial observer, one can use Minkowski modes as well, but expressed in terms of noninertial Fermi coordinates via (1). This is what is effectively done in (16) and (19) for a quantum case.

The algebra of quantum fields does not depend on the choice of a spacelike hypersurface on which equal-time commutation relations are imposed [28]. However, the knowledge of the algebra does not yet determine the physical system; one also needs to specify the *representation*. The Rindler and the Minkowski quantizations can be viewed as two inequivalent representations of the field algebra [29, 30, 31]. Not all possible representations need to be realized in nature. Just as Poincare covariance related to Minkowski spacetime does not imply that tachyons or higher-spin particles exist, the general covariance does not imply that Rindler particles exist. If the Unruh effect is viewed as in Sec. 5 and even if both types of particles exist separately, then, contrary to what is often attempted, the Rindler quantization cannot serve as an equivalent description of the Unruh effect. All known experiments can be explained in terms of Minkowski particles, so it seems that only the Minkowski representation is realized in nature.

There is also a controversy in the literature on whether the event horizon plays an essential role for understanding the Unruh effect. We are now able to give a definite answer to this question. If the Unruh effect is treated via the Rindler quantization, then the event horizon is essential. If it is treated via a model of a particle detector and the Minkowski quantization, then the event horizon is not essential. To repeat once more, these two treatments are not equivalent.

One could argue that rejecting all representations of the field algebra that are not equivalent to the Minkowski quantization also excludes quantizations based on the replacement of the Fourier integral by a Fourier sum, which would be in contradiction with the existence of the Casimir effect (see [18] for a review). However, at the fundamental level, the Casimir effect is a nonperturbative effect resulting from complicated field *interactions*, while

its description in terms of free fields that vanish on the boundary is only an effective, approximative description. Standard treatments of the particle production by moving mirrors ([18] and references therein) should also be understood in a similar way.

The existence of a preferred representation implies the existence of a preferred time, the one with respect to which the positive and the negative frequencies, and therefore the lowering and the raising operators, are defined. It is important to emphasize that this preferred time serves as a tool for the choice of the Hilbert space, i.e., the representation of the field algebra, but it does not violate the general covariance. It is not difficult to accept that in flat spacetime the time of an inertial observer has a privileged role (the quantizations based on different Lorentz times are equivalent). However, in curved spacetime, the quantizations based on Fermi coordinates of different inertial observers are not equivalent. How to choose the preferred time in general?

For many theorists, who believe that the fundamental laws of nature should be highly symmetrical, it is hard to believe that a preferred time could exist. Yet, it is an observational fact that a preferred time *does* exist, the one which is related to the homogeneity and isotropy of the Universe, as well as to the cosmological time arrow, which also seems to coincide with all other time arrows [32]. The time arrow is related to the initial condition of the Universe [33, 34]. In [34] it is proposed that quantization, i.e., the equal-time commutation relations, can also be viewed as an initial condition. The formalism of quantum field theory requires a choice of a special time, even when functional-integral techniques are employed [34]. In some interpretations of quantum mechanics it is even more explicit that a preferred time must exist. For example, if one proposes that wave functions collapse instantaneously, one must specify the time to which the instantaneousness refers. One cannot propose that this refers to the Fermi time of the measuring apparatus, because the measuring apparatus can be a large object, so each part of it may have a different time. In the de Broglie–Bohm interpretation of quantum field theory [35], the fundamental, deterministic equations of motion possess a preferred time and are manifestly noncovariant with respect to general coordinate transformations. In the nongeometrical interpretation of gravity, which may be the proper approach to quantum gravity, the choice of a preferred coordinate frame is also unavoidable [36]. The fixation of a coordinate frame before the quantization also resolves the problem of time in quantum gravity [37].

Our discussion strongly suggests that there *is* a preferred coordinate frame, but we still do not know *which* coordinate frame is the right one. Ultimately, this question should be answered experimentally. However, it is natural to expect that there is only one preferred time, i.e., that the time related to the quantization is the same time that is relevant to cosmology and the time arrow. This naturally leads to the conclusion that the right coordinates could be the normal Gauss coordinates, for which the metric takes the form

$$ds^2 = dt^2 - g_{ij}dx^i dx^j . \quad (22)$$

These coordinates are a generalization of Minkowski coordinates to curved spacetime. These also correspond to the simplest possible choice of coordinates in the Arnowitt-Deser-Misner approach to canonical gravity [38]. It seems that these coordinates could allow a global quantization for the whole Universe. For example, if the topology of the Universe is $\mathbf{R} \times S^3$, where \mathbf{R} represents the time, then one needs at least two regions with different coordinates.

However, these two coordinate frames may be related by a transformation of the form $t' = t$, $x'^i = f^i(x^1, x^2, x^3)$, which does not influence the time coordinate, which is essential for quantization. The Gauss coordinates are also appropriate for a global quantization in a black-hole background, because the metric is given by [39]

$$ds^2 = dt^2 - \frac{2M}{r_s} dr^2 - r_s^2 d\Omega^2, \quad (23)$$

where

$$r_s = (9M/2)^{1/3}(r - t)^{2/3} \quad (24)$$

is the Schwarzschild radial coordinate, so (23) does not possess any coordinate singularity, but only the genuine singularity at $r_s = 0$.

We do *not* require that the representations at different values of the *preferred* time should be equivalent. Therefore, when g_{ij} in (23) depends on t , the particle creation is possible. This allows the particle creation by the Universe expansion, but does not allow the Unruh effect via the Rindler quantization. It would be interesting to study Hawking radiation starting from the quantization with respect to the time t in (23), but the standard prediction of the thermal spectrum [40] is unlikely to be altered, because, if the radiation is viewed merely as particles that escaped from a collapsing body just before the event horizon was formed [18], the thermal spectrum can be understood even with classical physics [41, 42].

The preferred time leads to the preferred representation of the field algebra and therefore to the unambiguous formal definition of particles in terms of raising and lowering operators. There is also a simple, operational argument, why the number of real particles in a given state does not depend on the motion of the observer. Quantum detectors may respond to real particles as well as to “virtual particles” (or, better to say, to quantum fluctuations). On the other hand, classical detectors respond only to real particles. We believe that it is quite clear that two classical detectors will detect the same number of particles, irrespective of their motion (of course, providing that the trajectory of each particle crosses the trajectory of each detector once and only once). If we define radiation as emission of real particles, in this way we also confirm the conclusion of Sec. 4 that radiation does not depend on the observer.

8 Conclusion

For a given coordinate frame, values of covariant fields are mathematically determined at all spacetime points. However, a local observer can measure only the values at the point of his own position. By exploring the coordinate transformation between two Fermi frames, we have shown that two observers, having the same instantaneous position and velocity, will observe the same values of covariant fields at their common instantaneous position, even if they have different instantaneous accelerations. In particular, this implies that in classical physics the notion of radiation is observer independent. Consequently, an inertial charge in flat spacetime does not radiate from the point of view of an accelerated observer. However, a “freely” falling charge in *curved* spacetime does not move along a geodesic, so it *does* radiate (as seen by any observer).

The Rindler-quantization approach and the “particle-detector” approach to the Unruh effect are not equivalent. The Rindler quantization corresponds to a representation of the field algebra which is not equivalent to the Minkowski quantization, so the Rindler quantization should be rejected. There is a strong evidence that a preferred coordinate frame must exist, which allows an unambiguous formal definition of the particle number in curved spacetime. The response of classical particle detectors leads to an operational argument that the particle number does not depend on the observer.

Our discussion suggests that the fundamental problems of cosmology and quantum field theory are closely related. However, the concept of particle in quantum field theory is still far from being completely clear, even in Minkowski spacetime. For a clearer picture, we believe that it is essential to have a better understanding of nonperturbative effects, density-matrix decoherence, and the wave-function collapse.

Acknowledgment

This work was supported by the Ministry of Science and Technology of the Republic of Croatia under Contract No. 00980102.

References

- [1] H. Bondi and T. Gold, Proc. Roy. Soc. **A229**, 416 (1955).
- [2] F. Rohrlich, Nuovo Cim. **21**, 811 (1961).
- [3] F. Rohrlich, Ann. Phys. **22**, 169 (1963).
- [4] R. A. Mould, Ann. Phys. **27**, 1 (1964).
- [5] A. Kovetz and G. E. Tauber, Am. J. Phys. **37**, 382 (1969).
- [6] D. G. Boulware, Ann. Phys. **124**, 169 (1980).
- [7] S. Parrott, preprint gr-qc/9303025.
- [8] D. Lynden-Bell, J. Bičák, and J. Katz, Ann. Phys. **271**, 1 (1999).
- [9] M. Pauri and M. Vallisneri, preprint gr-qc/9903052, to appear in Found. Phys.
- [10] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
- [11] H. Nikolić, preprint gr-qc/9904078.
- [12] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco 1973).
- [13] A. Gupta and T. Padmanabhan, Phys. Rev. D **57**, 7241 (1998).
- [14] B. S. DeWitt and R. W. Brehme, Ann. Phys. **9**, 220 (1960).

- [15] J. M. Hobbs, *Ann. Phys.* **47**, 141 (1968).
- [16] T. C. Quinn and R. M. Wald, *Phys. Rev. D* **56**, 3381 (1997).
- [17] P. Meyer, *Phys. Rev. D* **18**, 609 (1978).
- [18] N. D. Birrell and P. C. W. Davies, *Quantum fields in curved space* (Cambridge Press, NY, 1982).
- [19] T. Padmanabhan, *Class. Quant. Grav.* **2**, 117 (1985).
- [20] J. Audretsch and R. Müller, *Phys. Rev. A* **50**, 1755 (1994).
- [21] K. Srinivasan, L. Sriramkumar, and T. Padmanabhan, *Int. J. Mod. Phys. D* **6**, 607 (1997); K. Srinivasan, L. Sriramkumar, and T. Padmanabhan, *Phys. Rev. D* **56**, 6692 (1997).
- [22] T. H. Boyer, *Phys. Rev. D* **21**, 2137 (1980).
- [23] V. D. Mur, B. M. Karanov, and V. S. Popov, *Zh. Exp. Teor. Fiz.* **114**, 798 (1998).
- [24] K.-P. Marzlin and J. Audretsch, *Phys. Rev. D* **57**, 1045 (1998).
- [25] W. G. Unruh and R. M. Wald, *Phys. Rev. D* **29**, 1047 (1984).
- [26] J. Audretsch and R. Müller, *Phys. Rev. D* **49**, 6566 (1994).
- [27] L. Sriramkumar and T. Padmanabhan, preprint gr-qc/9903054.
- [28] H. K. Urbantke, *Nuovo Cim.* **63B**, 203 (1969).
- [29] S. A. Fulling, *Phys. Rev. D* **7**, 2850 (1973).
- [30] U. H. Gerlach, *Phys. Rev. D* **40**, 1037 (1989).
- [31] A. M. Fedotov, V. D. Mur, N. B. Narozhny, V. A. Belinskii, and B. M. Karanov, *Phys. Lett. A* **254**, 126 (1999).
- [32] H. Nikolić, preprint physics/9812006.
- [33] S. W. Hawking, R. Laflamme, and G. W. Lyons, *Phys. Rev. D* **47**, 5342 (1993).
- [34] H. Nikolić, preprint gr-qc/9901045.
- [35] P. R. Holland, *Phys. Rep.* **224**, 95 (1993).
- [36] H. Nikolić, *Gen. Rel. Grav.* **31**, 1211 (1999).
- [37] C. J. Isham, in *Integrable Systems, Quantum Groups and Quantum Field Theories*, edited by L. A. Ibort and M. A. Rodriguez (Kluwer Academic Publishers, London, 1993); preprint gr-qc/9210011.

- [38] R. Arnowitt, S. Deser, and C. W. Misner, in *Gravitation: an introduction to current research*, edited by L. Witten (John Wiley & Sons, New York, 1963).
- [39] C. Møller, *The Theory of Relativity* (Oxford, New York, 1952).
- [40] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).
- [41] K. Srinivasan and T. Padmanabhan, preprint gr-qc/9812087.
- [42] M. Nouri-Zonoz and T. Padmanabhan, preprint gr-qc/9812088.